Period 1, Feb 10, 2025

$T = 2x^2$

$dT = 4x$

c) If $f(x) = \int_{5}^{2x^2} \sqrt{t^2+1}\, dt$, then $f'(-1)$ is ___________.

$F'(x) = \left(\sqrt{(2x^2)^2+1}\right)4x - 0$

$F'(x) = \left(\sqrt{4x^4+1}\right)4x - 0$

$F'(-1) = \sqrt{4(-1)^4+1} \cdot 4(-1) = \sqrt{5} \cdot -4 = -4\sqrt{5}$

$T = x$

$dT = dx$

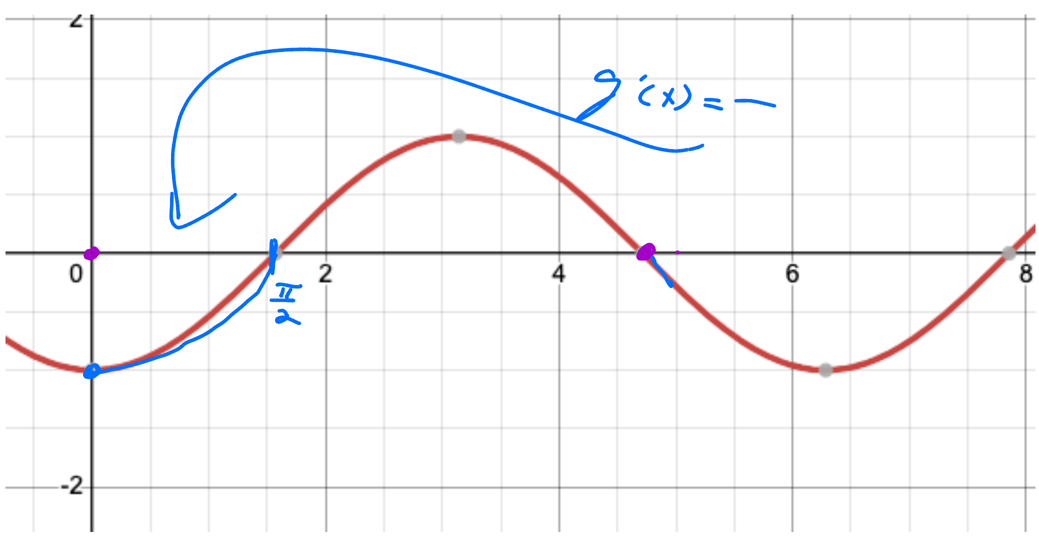
5. Suppose $g(x) = \int_0^x \sin\left(t - \frac{\pi}{2}\right) dt$ for $0 \le t \le \frac{3\pi}{2}$. On which interval is g is decreasing.

$g'(x) = \sin(x - \frac{\pi}{2})$

around 5 a little less

$- = g'(x) = $ slope of $g(x)$

$(0, \frac{\pi}{2})$



3. An object in rectilinear motion is moving along a horizontal line with velocity $v(t) = 3t^2 - 6t$ (in meters per second). If at time $t = 1$, the object is 2 m from the origin, what is its position at $t = 4$.

$V(T) = 3T^2 - 6T = 3T(T-2)$

Stopped $T=0$ and $T=2$

$S(T) = \int V(T)\,dT = \int (3T^2 - 6T)\,dT = T^3 - 3T^2 + C$

$S(1) = 2 = 1^3 - 3(1)^2 + C \Rightarrow 1 - 3 + C = 2 \Rightarrow C = 4$

$S(T) = T^3 - 3T^2 + 4$

$S(4) = 4^3 - 3(4)^2 + 4 = 64 - 48 + 4 = 20$

or

$\int_1^4 (3T^2 - 6T)\,dT = 4^3 - 3(4)^2 - [1^3 - 3(1)^2] = 64 - 48 - 1 + 3 = 18$

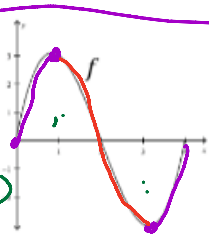
$S(4) = \left[\int_1^4 (3T^2 - 6T)\,dT\right] + 2 = 18 + 2 = 20$

6. The graph of f is given below.

Slope of $g(x)$

$g(x)$ increasing $(0,2)$

$g(x)$ is decreasing $(2,4)$



Slope of $F(x)$ is positive

Slope of $F(x)$ is negative

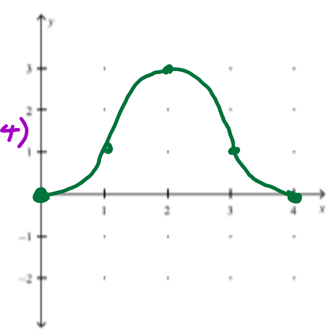
Let $g(x) = \int_0^x f(t)\,dt$ for $0 \le x \le 4$.

a) When is g increasing? Decreasing? Justify your answer. Find the x-value of the relative extremum of g.

b) When is g concave up? Concave down? Justify your answer.

$g''(x) = F'(x) = + =$ Slope of $F(x)$ concave up $(0,1) \cup (3,4)$

$g''(x) = F'(x) = - =$ Slope of $F(x)$ concave down $(1,3)$



c) Use the information above to sketch g on the axis provided (for $0 \le x \le 4$)

$$\int (x+2)^2\,dx =$$

$$\int (x^2 + 4x + 4)\,dx$$

$$\frac{1}{3}x^3 + 2x^2 + 4x + C$$

$$\int (\underbrace{x+2}_{})^{100}\,dx =$$

$u = x+2$

$du = dx$

$$\int u^{100}\,du$$

$$\frac{1}{101}u^{101} + C$$

$$\frac{1}{101}\cdot(x+2)^{101} + C$$

Outside function

Composition of Functions

$$\int f'(\underbrace{g(x)}_{u})\underbrace{g'(x)dx}_{du} = f(\underbrace{g(x)}_{u}) + C$$

inside function

Derivative of inside function

$$u = g(x)$$
$$du = g'(x)dx$$

When deciding what to use for *u*, choose functions in this order: **LIPET**
Logs,
Inverse Trig,
Polynomials,
Exponential,
Trig

$$\int f'(u)du = f(u) + C$$

Consider this "Anti-Chain Rule"

$$\int x^2 dx = \frac{1}{3}x^3 + C$$

$$\int \sqrt{1+x^2} \cdot 2x\,dx = \int \sqrt{u} \cdot \cancel{2x} \cdot \frac{du}{\cancel{2x}}$$

$u = 1+x^2$

$du = 0 + 2x\,dx$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$a^{\frac{3}{2}} = a^{\frac{2}{2}} \cdot a^{\frac{1}{2}}$

$a^1 \cdot \sqrt{a}$

$\int \sqrt{u}\,du = \int u^{\frac{1}{2}}\,du = \frac{2}{3} u^{\frac{1}{2}+1=\frac{3}{2}} + C$

$\frac{2}{3} u^{\frac{3}{2}} + C$

$\frac{2(1+x^2)^{3/2}}{3} + C$

$\frac{2(1+x^2)\sqrt{1+x^2}}{3} + C$

$$\int \sqrt{4x-1}\,dx = \int \sqrt{u} \cdot \frac{du}{4} = \int \frac{1}{4} u^{\frac{1}{2}} du$$

$u = 4x-1$

$du = 4dx$

$\frac{du}{4} = dx$

$\frac{1}{4} \cdot \frac{2}{3} \cdot u^{\frac{1}{2}+1=\frac{3}{2}} + C$

$\frac{2}{12} u^{3/2} + C$

$\frac{1}{6} u^{3/2} + C$

$\frac{1}{6}(4x-1)^{3/2} + C = \frac{(4x-1)\sqrt{4x-1}}{6} + C$

$$\int \cos(7x+5)\ dx = \int \cos u \cdot \frac{du}{7} = \int \frac{1}{7}\cos u\, du$$

$u = 7x+5$

$du = 7dx$

$\frac{du}{7} = dx$

$\frac{1}{7}\sin u + C$

$\frac{1}{7}(\sin(7x+5)) + C$

$$\int x^2 \sin(x^3)\ dx = \int \cancel{x^2}\sin u \cdot \frac{du}{3\cancel{x^2}} = \int \frac{1}{3}\sin u\, du$$

$u = x^3$

$du = 3x^2 dx$

$\frac{du}{3x^2} = dx$

$\frac{1}{3}(-\cos u) + C$

$-\frac{1}{3}[\cos x^3] + C$

$-\frac{\cos x^3}{3} + C$

$$\int \sin^4 x \cdot \cos x\ dx$$

$u = \sin x$

$du = \cos x dx$

$\frac{du}{\cos x} = dx$

$$\int u^4 \cdot \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}} = \frac{1}{5} u^{4+1=5} + C = \frac{1}{5}(\sin^5 x) + C = \frac{1}{5}(\sin x)^5 + C$$

$$\int e^{2x}\,dx = \int e^{u} \cdot \frac{du}{2}$$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\frac{1}{2}\int e^{u}\,du$

$\frac{1}{2}e^{u} + c$

$\frac{1}{2}e^{2x} + C$

$$\int 5xe^{-x^2}\,dx$$

$u = -x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$\int 5\cancel{x} \cdot e^{u} \cdot \frac{du}{-2\cancel{x}} = \int -\frac{5}{2}e^{u}\,du$

$-\frac{5}{2}\int e^{u}\,du$

$-\frac{5}{2}e^{u} + c = -\frac{5}{2}e^{-x^2} + c$

$$\int \sec^4 3x \tan 3x\,dx = \int \sec^4 u \,\mathrm{Tan}\, u \cdot \frac{du}{3}$$

$3x = u$

$3dx = du$

$dx = \frac{du}{3}$

$L = \sec u$

$dL = \sec u \,\mathrm{Tan}\, u\,du$

$\frac{dL}{\sec u \,\mathrm{Tan}\, u} = du$

$\int \sec^4 u \cdot \mathrm{Tan}\, u \cdot \frac{du}{3}$

$\frac{1}{3}\int \sec^4 u \,\mathrm{Tan}\, u\,du$

$\frac{1}{3}\int L^{\cancel{4}\,3} \cdot \cancel{\mathrm{Tan}\, u} \cdot \frac{dL}{\cancel{L} \cdot \cancel{\mathrm{Tan}\, u}}$

$\frac{1}{3}\int L^3\,dL = \frac{1}{3} \cdot \frac{1}{4} \cdot L^{3+1=4} + c$

$\frac{1}{12}\sec u^4 + c = \frac{1}{12}\sec^4 3x + C$

$\int \sec^4 u \,\mathrm{Tan}\, u \cdot \frac{du}{3}$

$\frac{1}{3}\int \sec^4 u \,\mathrm{Tan}\, u\,du$

$\frac{1}{3}\int \sec^{\cancel{4}\,3} u \cdot \cancel{\mathrm{Tan}\, u} \cdot \frac{dL}{\cancel{\sec u}\ \cancel{\mathrm{Tan}\, u}}$

$\frac{1}{3}\int L^3\,dL$

$$\int \frac{e^x}{e^x+4}\,dx$$

$u = e^x + 4$

$du = e^x\,dx$

$\frac{du}{e^x} = dx$

$\int \frac{\cancel{e^x}}{e^x+4} \cdot \frac{du}{\cancel{e^x}}$

$\int \frac{1}{u} \cdot du = \ln|u| + C$

$\ln(e^x+4) + C$

$- \frac{1}{2}\ln|8-2x| + C$

$$\int \frac{1}{8-2x}\,dx$$

$u = 8-2x$

$du = -2\,dx$

$\frac{du}{-2} = dx$

$\int \frac{1}{u} \cdot \frac{du}{-2} = -\frac{1}{2}\int \frac{1}{u}\,du$

$-\frac{1}{2}\ln|u| + C = -\frac{1}{2}\ln|8-2x| + C$

Example 8

$\ln x^3 = 3\ln x$

$\frac{1}{2}\ln|x^2+2x| + C$

$$\int \frac{x+1}{x^2+2x}\,dx$$

$u = x^2 + 2x$

$du = (2x+2)\,dx$

$\frac{du}{2x+2} = dx$

$\int \frac{x+1}{u} \cdot \frac{du}{2x+2}$

$\int \frac{\cancel{x+1}}{u} \cdot \frac{du}{2\cancel{(x+1)}} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x| + C = \ln|(x^2+2x)^{\frac{1}{2}}| + C$

$\ln|\sqrt{x^2+2x}| + C$

Solve: $\frac{dy}{dx} = \frac{1}{x \ln x}$

$\int dy = \int \frac{1}{x \ln x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$y = \int \frac{1}{x \ln x} dx$

$x\,du = dx$

$y = \int \frac{1}{\cancel{x} \cdot u} \cdot \cancel{x}\,du = \ln|u| + c = \ln|\ln x| + c$

Integration Rules for Exponential Functions to a base other than e

Let u be a differentiable function of x.

1) $\int a^x \, dx = \left(\frac{1}{\ln a}\right) a^x + C$

$\frac{d}{dx}[a^u] = \ln a \cdot a^u$

2) $\int a^u \, du = \left(\frac{1}{\ln a}\right) a^u + C$

Use this rule when you do **u-substitution**

1) $\int 5^{2x} \, dx$

$u = 2x$

2) $\int \sin 2x \, 5^{\cos 2x} \, dx$

$u = 2x \Rightarrow du = 2dx \Rightarrow \frac{du}{2} = dx$

$L = \cos u \Rightarrow dL = -\sin u \frac{du}{2}$

$$\int 5^{2x}\,dx \qquad u = 2x$$

$$du = 2\,dx$$

$$\int 5^u \cdot \frac{du}{2} \qquad \frac{du}{2} = dx$$

$$\frac{1}{2}\int 5^u\,du = \frac{1}{2}\left[\frac{1}{\ln 5}\cdot 5^u\right] + C$$

$$\frac{1}{2}\cdot\frac{1}{\ln 5}\cdot 5^{2x} + C = \frac{5^{2x}}{\ln 25} + C$$

$$\frac{1}{\ln 5^2}$$

$$\int \sin u\; 5^{\cos u}\cdot\frac{du}{2} \qquad \frac{dL}{-\sin u} = du$$

$$\frac{1}{2}\int \sin u \cdot 5^{\cos u}\cdot du$$

$$\frac{1}{2}\int \cancel{\sin u}\cdot 5^{\cos u}\cdot\frac{dL}{-\cancel{\sin u}}$$

$$-\frac{1}{2}\int 5^L\,dL = -\frac{1}{2}\left[\frac{1}{\ln 5}\right]\cdot 5^L + C$$

$$-\frac{1}{2}\cdot\frac{1}{\ln 5}\cdot 5^{\cos 2x} + C$$

- $\int \tan u\,du = -\ln|\cos u| + C = \ln|\sec u| + C$
- $\int \sec u\,du = \ln|\sec x + \tan x| + C$
- $\int \cot u\,du =$ (left for an exercise for student)
- $\int \csc u\,du =$(left for an exercise for student)

Integrals of the Six Basic Trigonometric Functions

$$\int \sin u\,du = -\cos u + C$$

$$\int \cos u\,du = \sin u + C$$

$$\int \tan u\,du = -\ln\left|\cos u\right| + C$$

$$\int \cot u\,du = \ln\left|\sin u\right| + C$$

$$\int \sec u\,du = \ln\left|\sec u + \tan u\right| + C$$

$$\int \csc u\,du = -\ln\left|\csc u + \cot u\right| + C$$

On the quiz, you will be asked to *derive*
tan u, cot u, sec u, or csc u

Recall that from the rules for differentiation,

Derivatives of an Inverse Trigonometric Function

1) $\frac{d}{dx}[arc\sin u] = \frac{u'}{\sqrt{1-u^2}}$

2) $\frac{d}{dx}[arc\cos u] = \frac{-u'}{\sqrt{1-u^2}}$

3) $\frac{d}{dx}[arc\tan u] = \frac{u'}{1+u^2}$

4) $\frac{d}{dx}[arc\cot u] = \frac{-u'}{1+u^2}$

5) $\frac{d}{dx}[arc\sec u] = \frac{u'}{|u|\sqrt{u^2-1}}$

6) $\frac{d}{dx}[arc\csc u] = \frac{-u'}{|u|\sqrt{u^2-1}}$

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

$$\int \frac{dx}{\sqrt{e^{2x} - 1}}$$

$u = e^x \Rightarrow u^2 = e^{2x}$

$du = e^x dx$

$\frac{du}{e^x} = dx$

$\frac{du}{u} = dx$

$$\int \frac{dx}{\sqrt{u^2 - 1^2}} = \int \frac{1}{\sqrt{u^2 - 1^2}} \cdot \frac{du}{u}$$

$$\int \frac{du}{u\sqrt{u^2 - 1}} = \frac{1}{1} \operatorname{arcsec} \frac{|u|}{1} + C$$

$\operatorname{arcsec} |e^x| + C$

$\operatorname{arcsec} e^x + C$